1 Precheck: Number Representation

1.1 Depending on the context, the same sequence of bits may represent different things.

1.2 It is possible to get an overflow error in Two's Complement when adding numbers of opposite signs.

1.3 If you interpret an n-bit Two's complement number as an unsigned number, negative numbers would be smaller than positive numbers.

1.4 If you interpret an n-bit Bias notation number as an unsigned number (assume there are negative numbers for the given bias), negative numbers would be smaller than positive numbers.

1.5 We can represent fractions and decimals in our given number representation formats (unsigned, biased, and Two's Complement).

2 Unsigned and Signed Integers

2.1 Convert the following numbers from their initial radix into the other two common radices:

(a) `0b10110011`

(b) `0`

(c) `437`

(d) `0x0123`

2.2 Convert the following numbers from hex to binary:

(a) `0xD3AD`

(b) `0x7EC4`

2.3 Assuming an 8-bit integer and a bias of −127 where applicable, what is the largest integer for each of the following representations? What is the result of adding one to that number?

(a) Unsigned

(b) Biased

(c) Two's Complement

2.4 How would you represent the numbers 0, 1, and −1? Express your answer in binary and a bias of −127 where applicable.

(a) Unsigned

(b) Biased

(c) Two's Complement

2.5 How would you represent the numbers 17 and −17? Express your answer in binary and a bias of −127 where applicable.

(a) Unsigned

(b) Biased

(c) Two's Complement

2.6 What is the largest integer that can be represented by *any* encoding scheme that only uses 8 bits?

2.7 Prove that that $x + \bar{x} + 1 = 0$, where $\bar{x}$ is obtained by inverting the bits of x in binary.

3 Arithmetic and Counting

3.1 Compute the decimal result of the following arithmetic expressions involving 6-bit Two's Complement numbers as they would be calculated on a computer. Do any of these result in an overflow? Are all these operations possible?

(a) `0b011001 - 0b000111`

(b) `0b100011 + 0b111010`

(c) `0x3B + 0x06`

(d) `0xFF - 0xAA`

(e) `0b000100 - 0b001000`

3.2 How many distinct numbers can the following schemes represent? How many distinct positive numbers?

(a) 10-bit unsigned

(b) 8-bit Two's Complement

(c) 6-bit biased, with a bias of -30

(d) 10-bit sign-magnitude